On Mathematics of Seki Takakazu

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Seki Takakazu

Seki Takakazu (?–1708)

- Seki Takakazu was a great mathematician of the Edo period.
- He broke free from the constraints of classical Chinese mathematics he had inherited and created a new mathematics.
- He argued that general theories were essential in mathematics, and developed general theories in areas such as the theory of equations and elementary number theory. However, his view of mathematics was never understood by the mathematicians of the Edo period who came after him.
- He was not a professional mathematician; he served as a mid-level bureaucrat, first in the Kofu domain and later in the Edo Shogunate.

Seki Takakazu

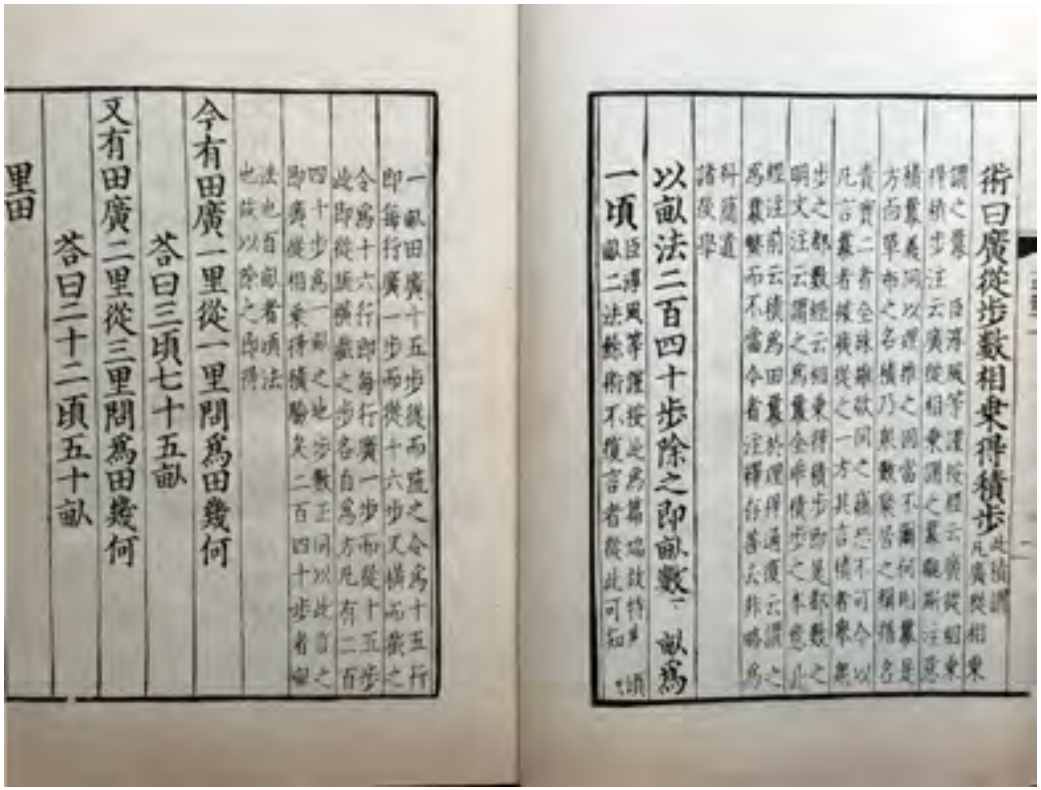
Seki Takakazu (?–1708)

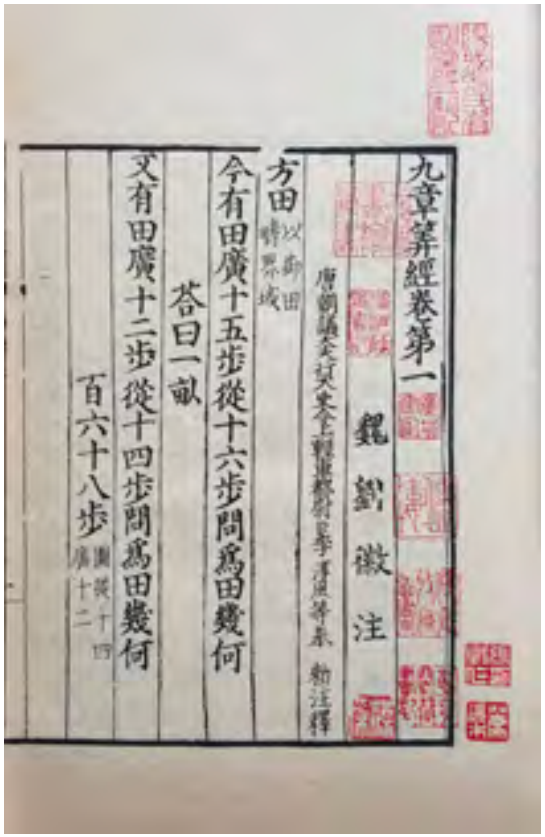
- Born as the second son of Uchiyama Nagaakira, a retainer of the shogunate.
- Later adopted into the Seki family of the Kofu Domain, stationed in Edo.
- 1704: Tokugawa Tsunatoyo, the daimyō of the Kofu Domain, became the heir apparent to Shogun Tsunayoshi. They moved to Nishinomaru in Edo Castle
- Seki Takakazu became a retainer of the shogunate, serving as head of the heir's personal treasury and wardrobe (Nishinomaru Onando Kumi-Gashira).
- Seki retired in 1706 and died of illness in 1708.
- 1709 Shogun Tsunayoshi died and Tsunatoyo succeeded him as the sixth shogun, Ienobu.

Classical Chinese Mathematics

- Classical Chinese mathematics developed primarily within the framework of *The Nine Chapters on the Mathematical Art*, written two thousand years ago and regarded as a canon.
- A collection of problems divided into nine chapters according to the methods used to solve them.
- *The Nine Chapters* : Problems, answers, and methods of solution.
- Many of these methods were presented as algorithms and were highly sophisticated.
- However, it lacked the concepts of **proof** and a **structural theoretical system** (**general theory**).

The Nine Chapters





Classical Chinese Mathematics

- Mathematics is a technique for solving problems.
- Liu Hui of Wei during the Three Kingdoms (3rd century) wrote a commentary on *The Nine Chapters*, in which he emphasized the importance of proof. However, his ideas were not adopted!
- Chapter 8 of *The Nine Chapters* deals with systems of linear equations. It presents an algorithm for solving systems of linear equations in a form similar to the modern use of matrices. However, general theories of systems of linear equations—such as Cramer's rule—as well as general theories of matrices and determinants never emerged in classical Chinese mathematics.

Classical Chinese Mathematics



A matrix represented using counting rod symbols can be found on the left page of *Suanxie Qimeng*.

Classical Chinese Mathematics

- Classical Chinese mathematics reached its peak during the Jin, Southern Song and Yuan dynasties. In particular, the "tianyuan shu" (the celestial element method) was developed during the Jin dynasty as a method of constructiong an equation on paper using counting rod symbols.
- The "tianyuan shu" was used to create the Shuishi calendar, but it was completely forgotten during the subsequent Ming dynasty. However, it was preserved in Korea.

Classical Chinese Mathematics

Seki learned classical Chinese mathematics from two books:

- *Yang Hui Sanfa*『楊輝算法』 Yang Hui (13th century, Southern Song)

  Published in China in 1397.

  Republished in Korea in 1433(癸丑).

  Seki wrote a manuscript that was a revised version of the *Yang Hui Sanfa* in 1673(癸丑).

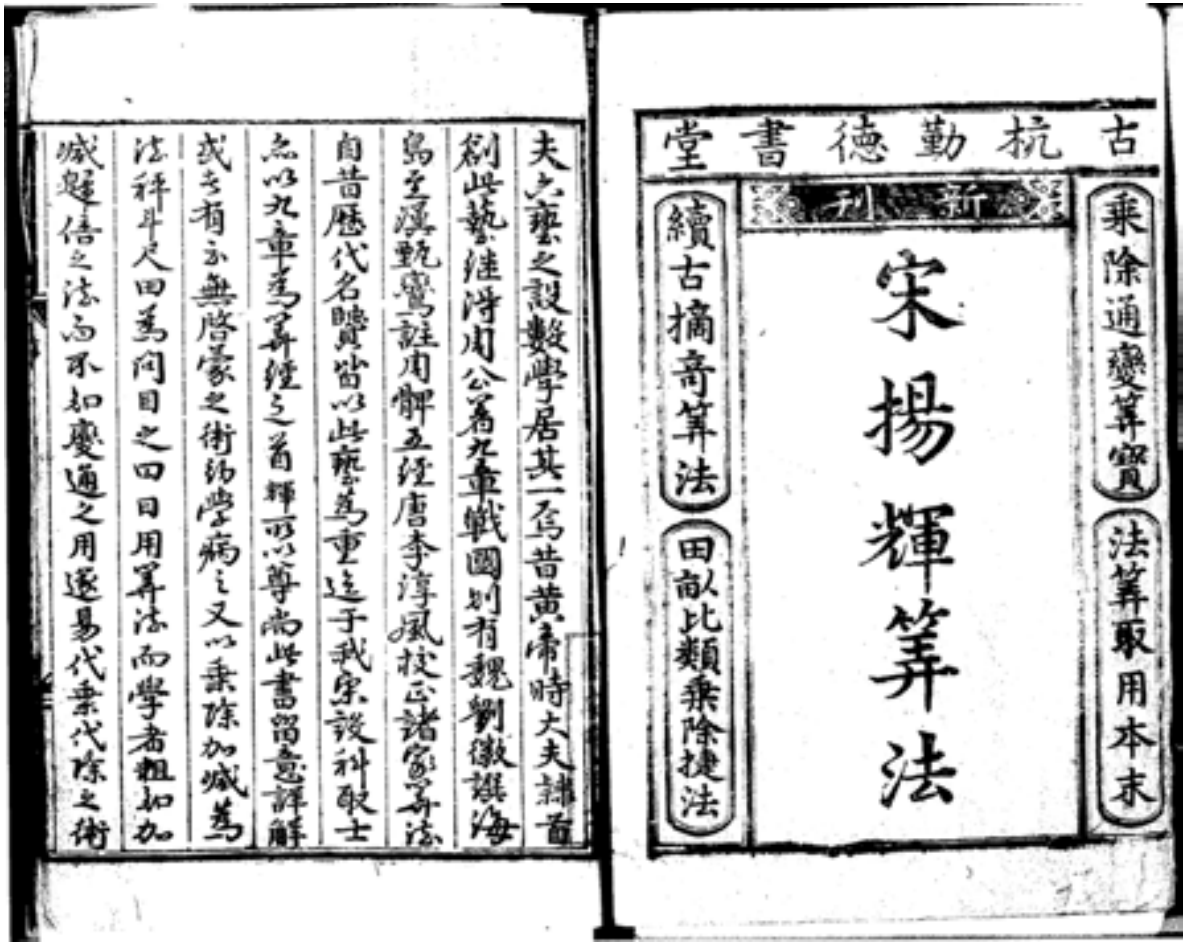
- Suanxue Qimeng 『算学啓蒙』

  Published in China in 1299.

  Republished in Korea during the reign of King Sejong(1419–1450).

  Based on the Korean version, it was republished in Japan in 1658.

Yang Hui Sanfa



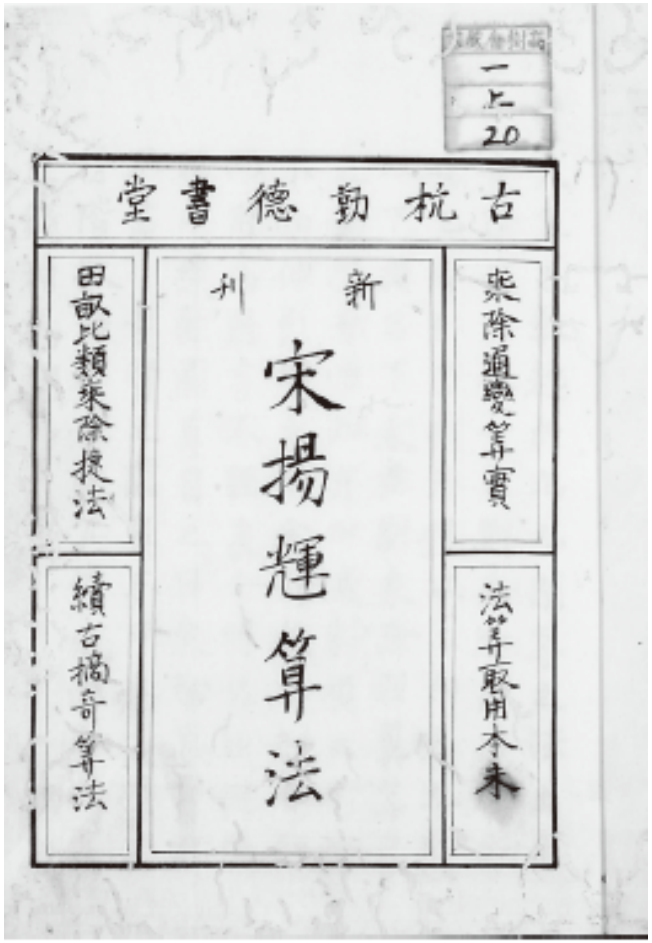
Korean version. Tsukuba University Library, Former Yoan-in collection.

Suanxue Qimeng



Korean version. Tsukuba University Library, Former Yoan-in collection.

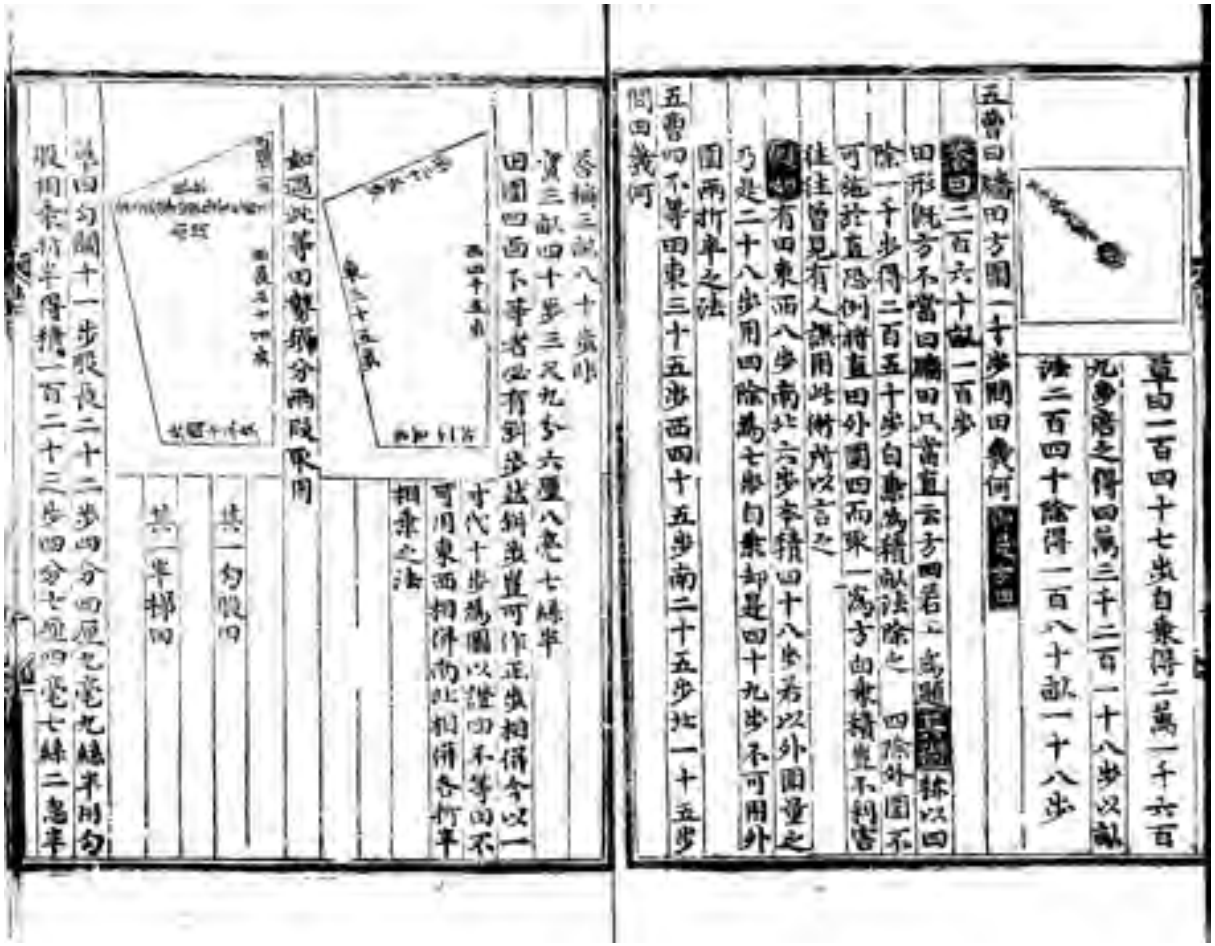
Seki's revised version of *Yang Hui Sanfa*





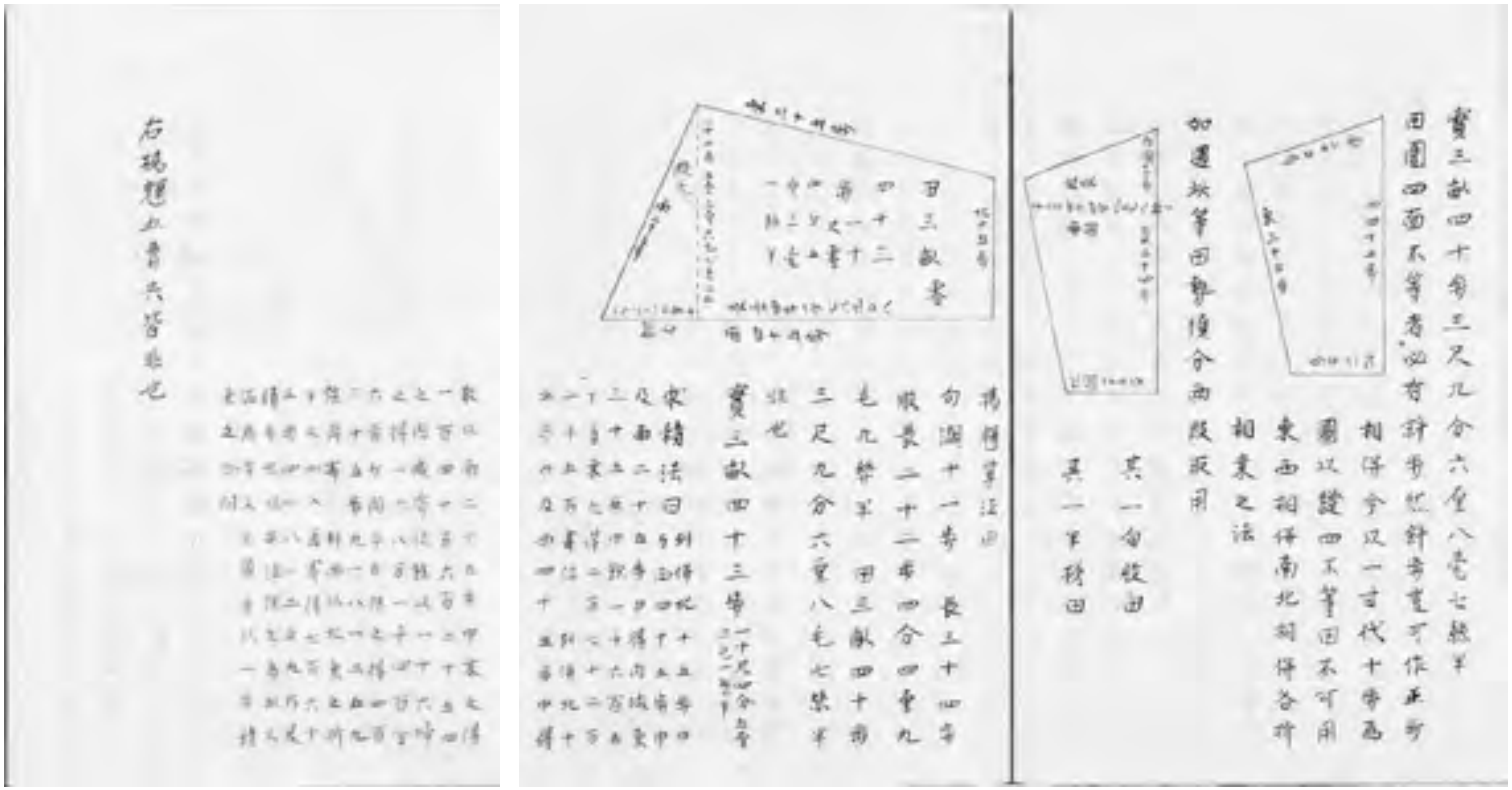
Koju-bunko, Former Yabuuchi collection.

Yang Hui Sanfa



Yang Hui 'corrected' an error in a solution from *Wucao suanfa*.

Seki's correction

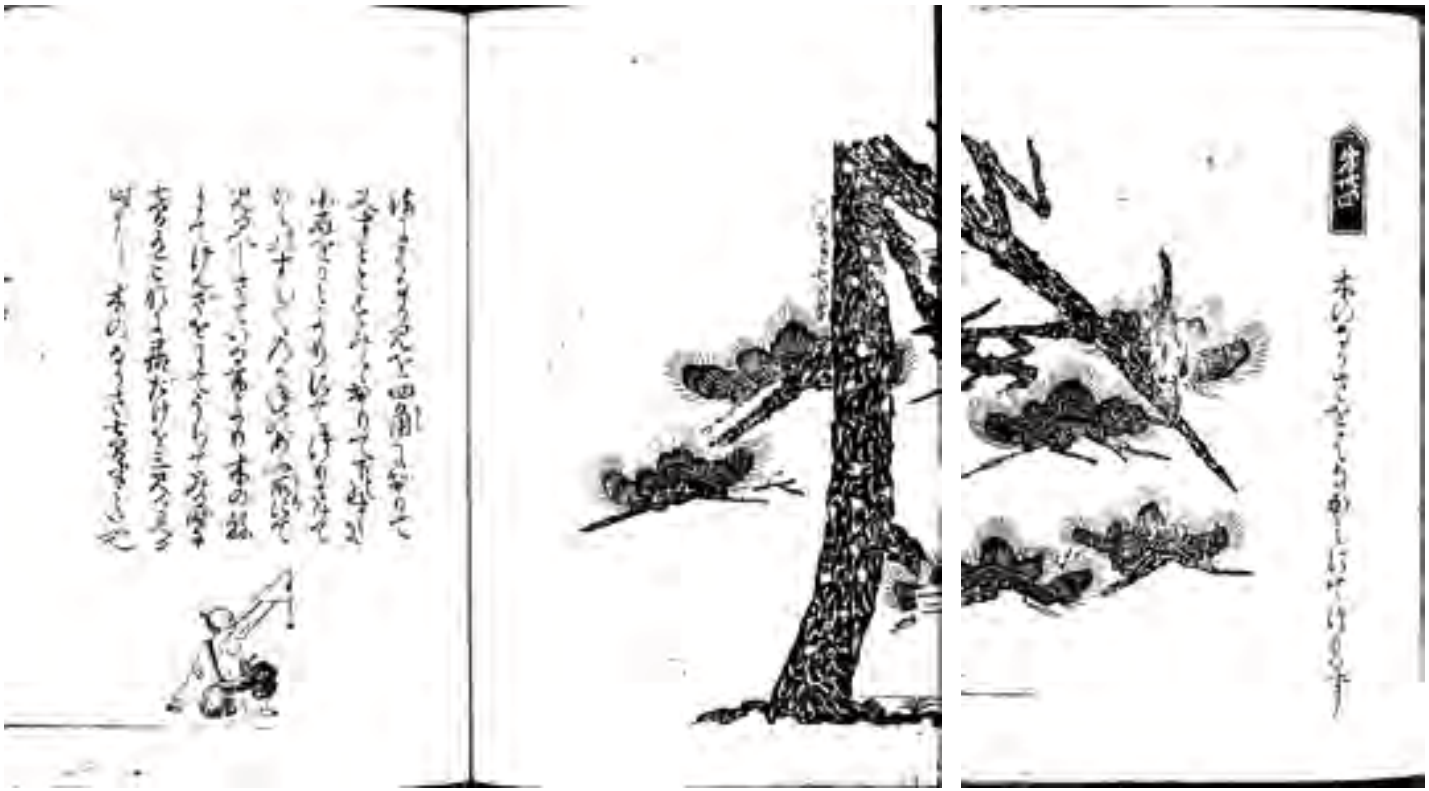


Koju-bunko

Japanese Mathematics before Seki

- The study of mathematic in the Edo period had begun with the publication of the *Jinkōki* by Yoshida Mitsuyoshi in1627.
- The *Jinkōki* was a textbook on the abacus, and its excellent content led to the circulation of many pirated editions.
- To compete with the pirated versions, Mitsuyoshi published revised editions in 1631, 1634 and 1641.
- The last edition published in 1641 played an important role in the later development of mathematics during the Edo period.

Yohsida Mitsuyoshi *Jinkōki* Yoshida Mitsuyoshi : *Jinkōki*, 1627



Mamakodate

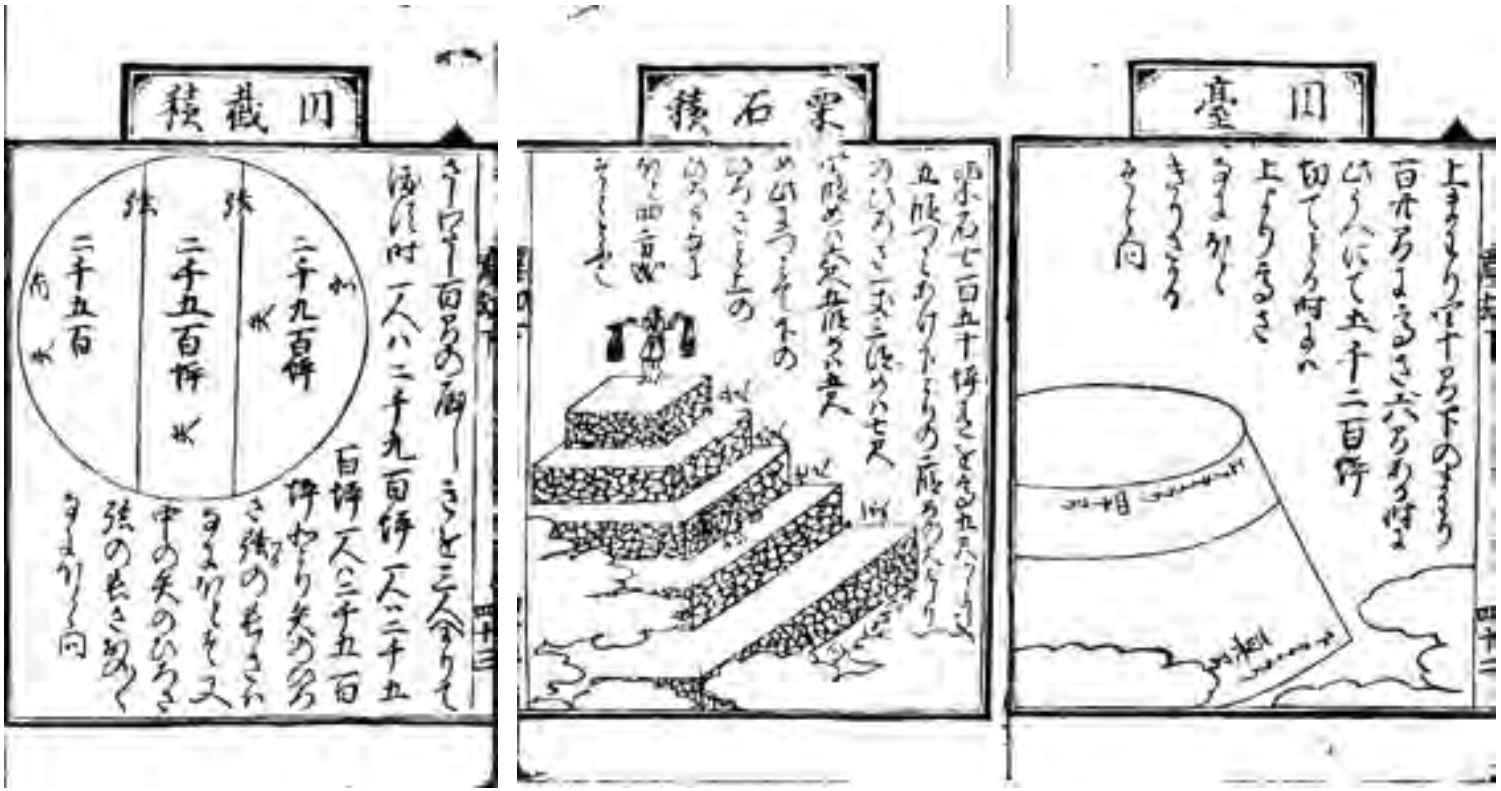




1631 edition 1634 edition (Tohoku University Library Hayshi Bunko0547)

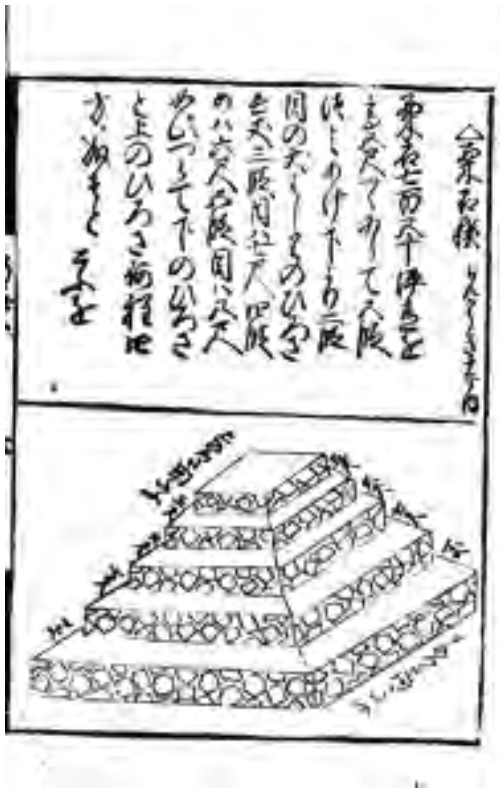
Jinkōki, 1641 edition

Unanswered problems: Problems 8– 10



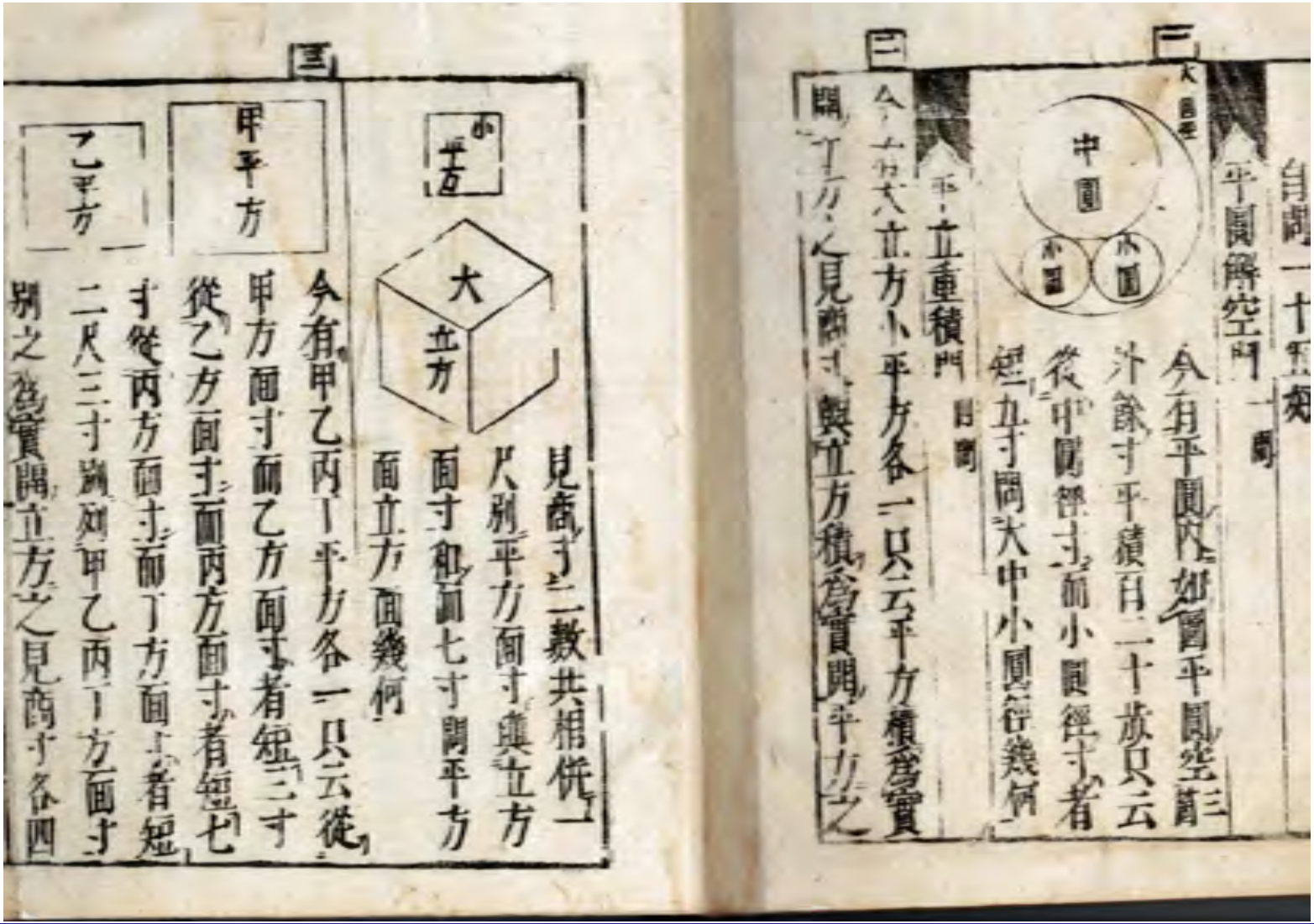
Yamada Masashige *Kaisanki* 1653



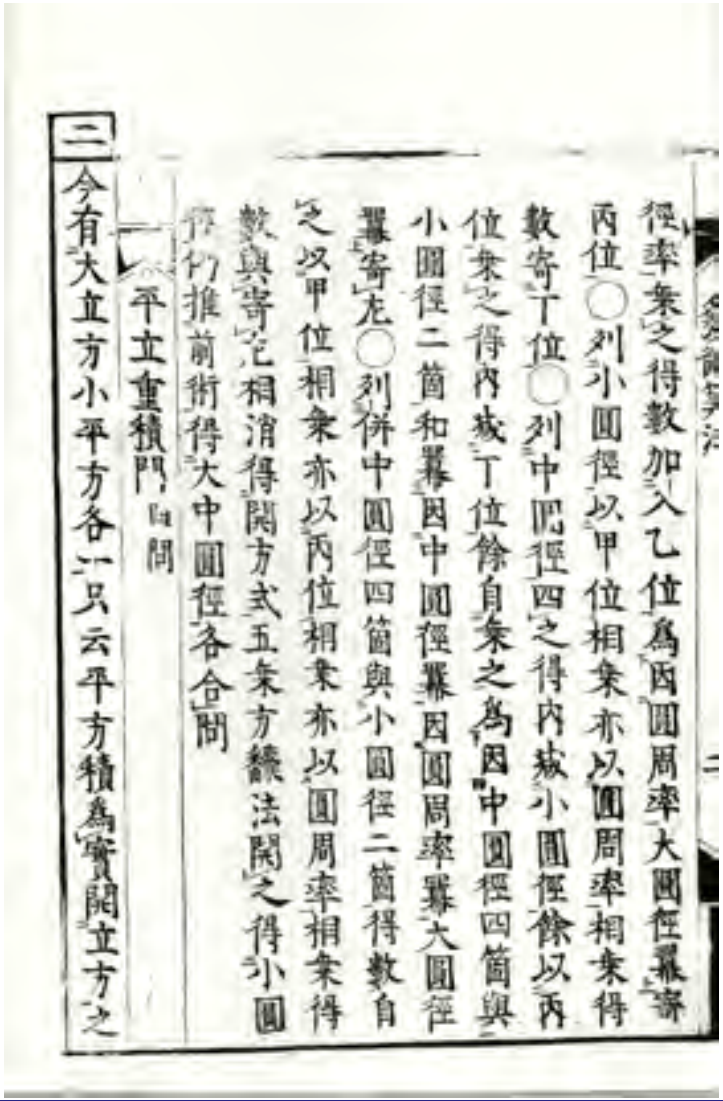


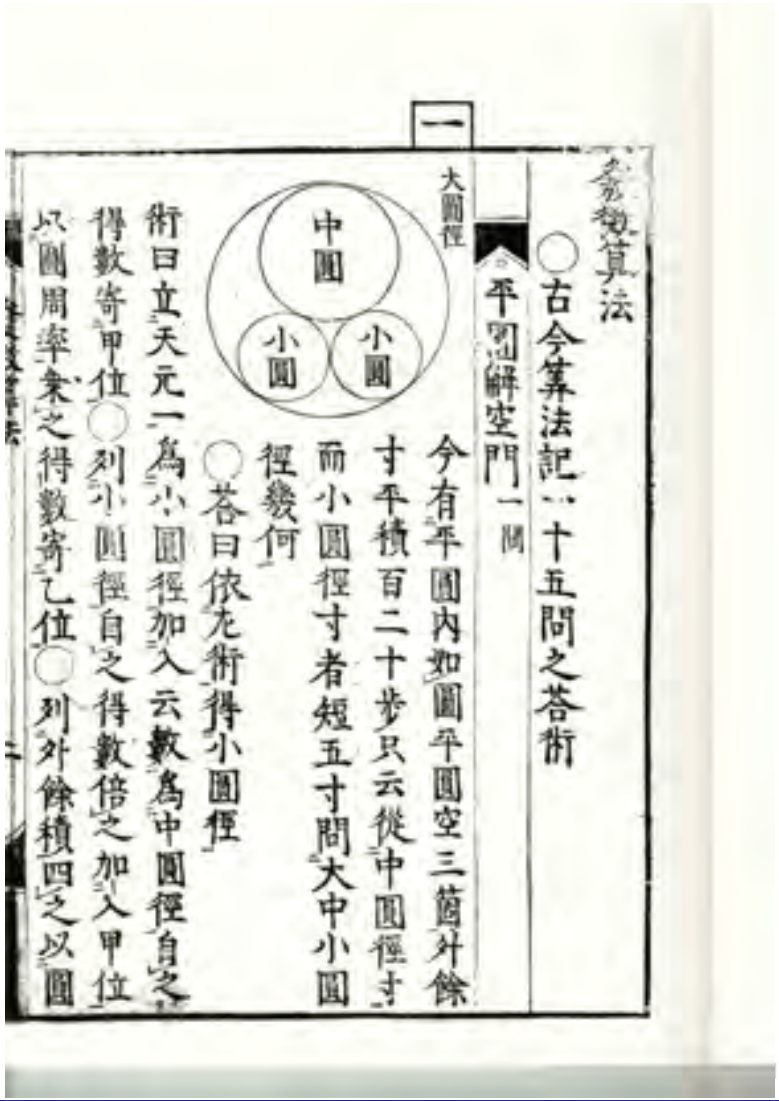


Sawaguchi Kazuyuki *Kokon Sanpo-Ki 1671*



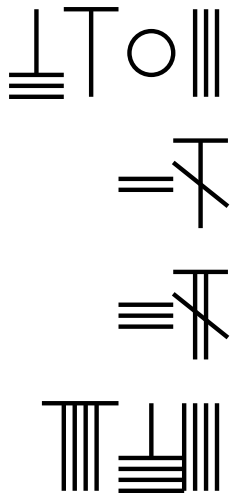
Seki Takakazu *Hatsubi Sanpo* 1674





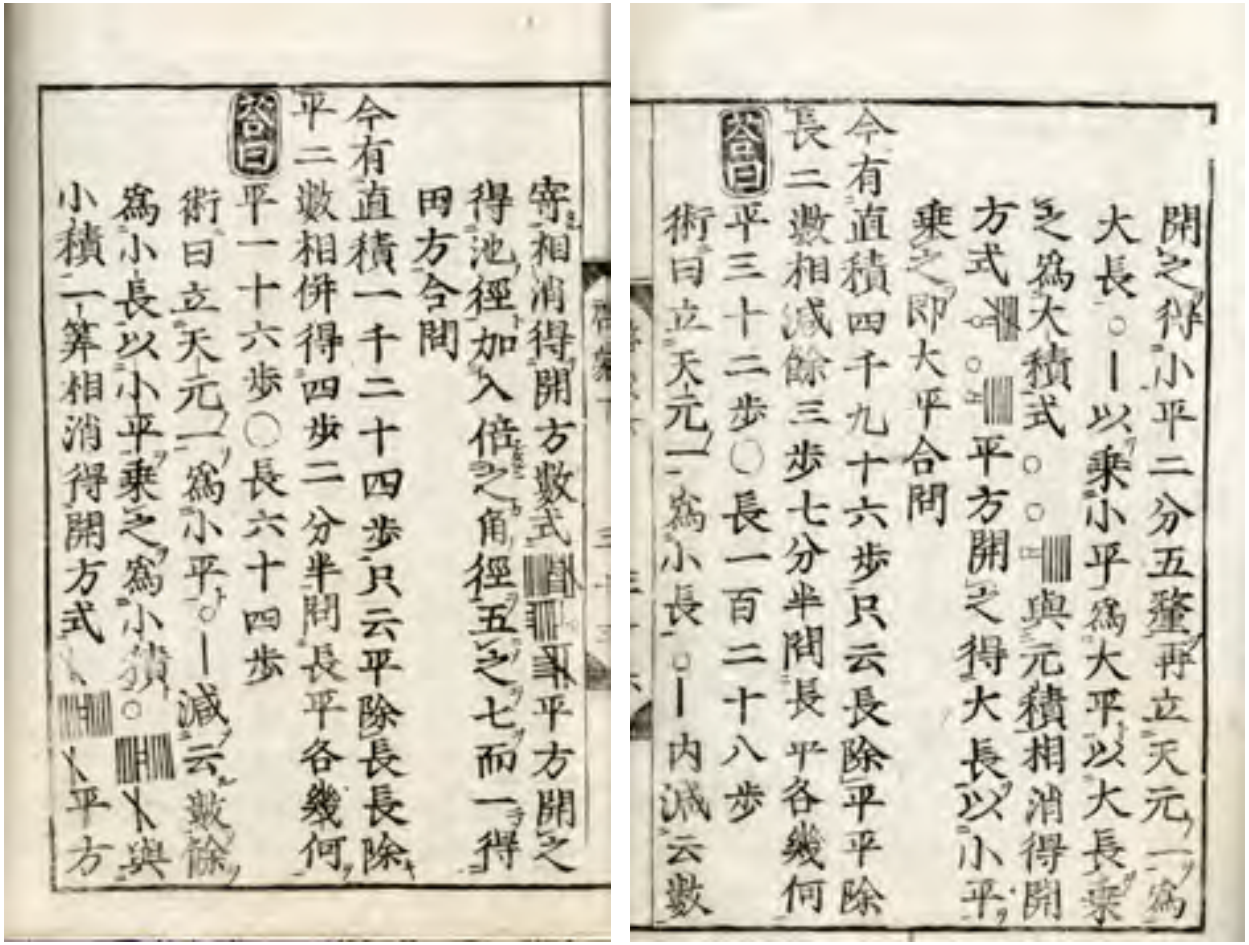
Tianyuan shu

Tianyuan shu(celestal element method)
$8603 - 26x - 37x^2 + 894x^3 = 0$



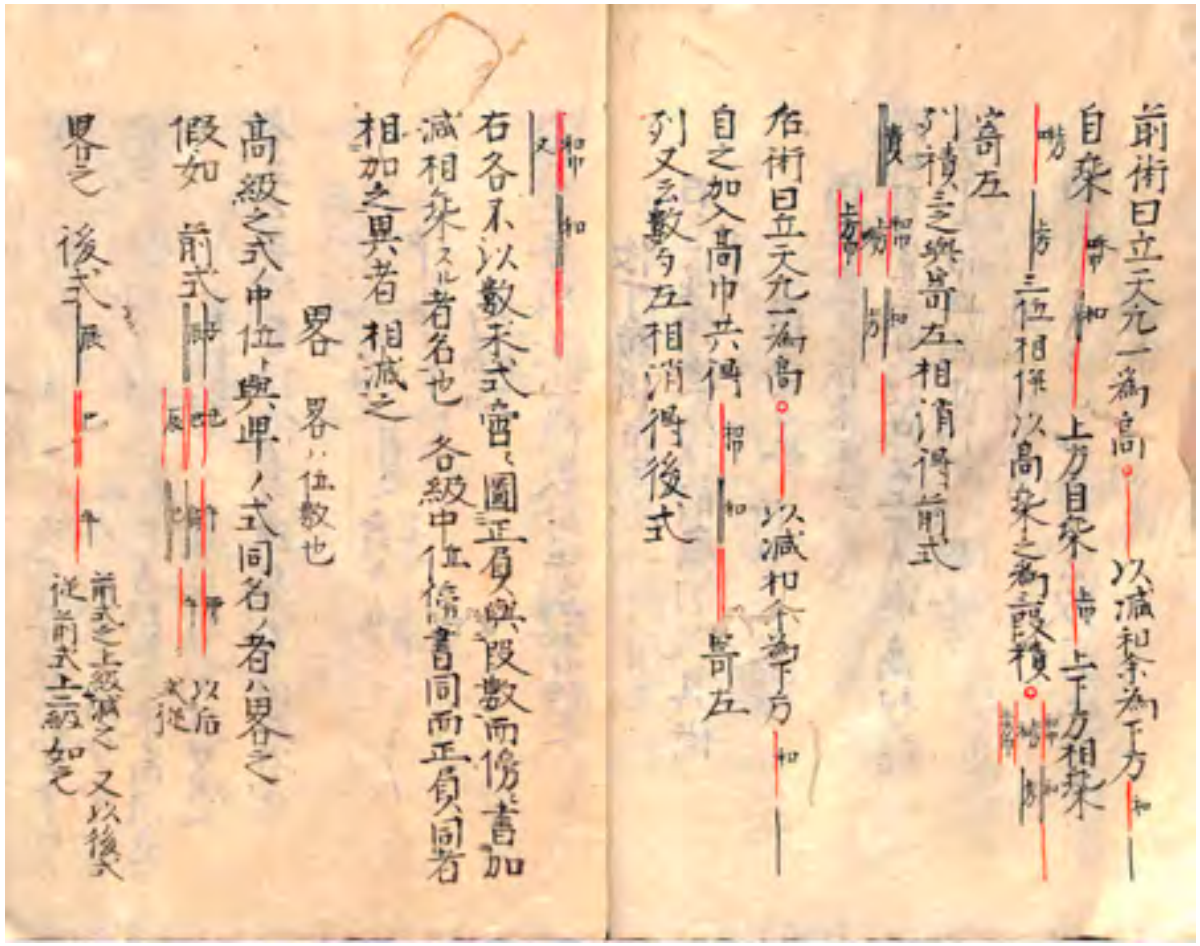
It can be expressed as an equation with one unknown.

Tianyuan shu



Suanxue Qimeng, Japanese edition 1651

Boshohō(side writing method)



Kaifukudai no Hō, Wasan no Yakata

Boshohō(side writing method)

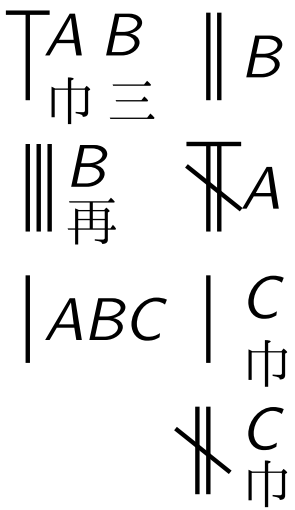
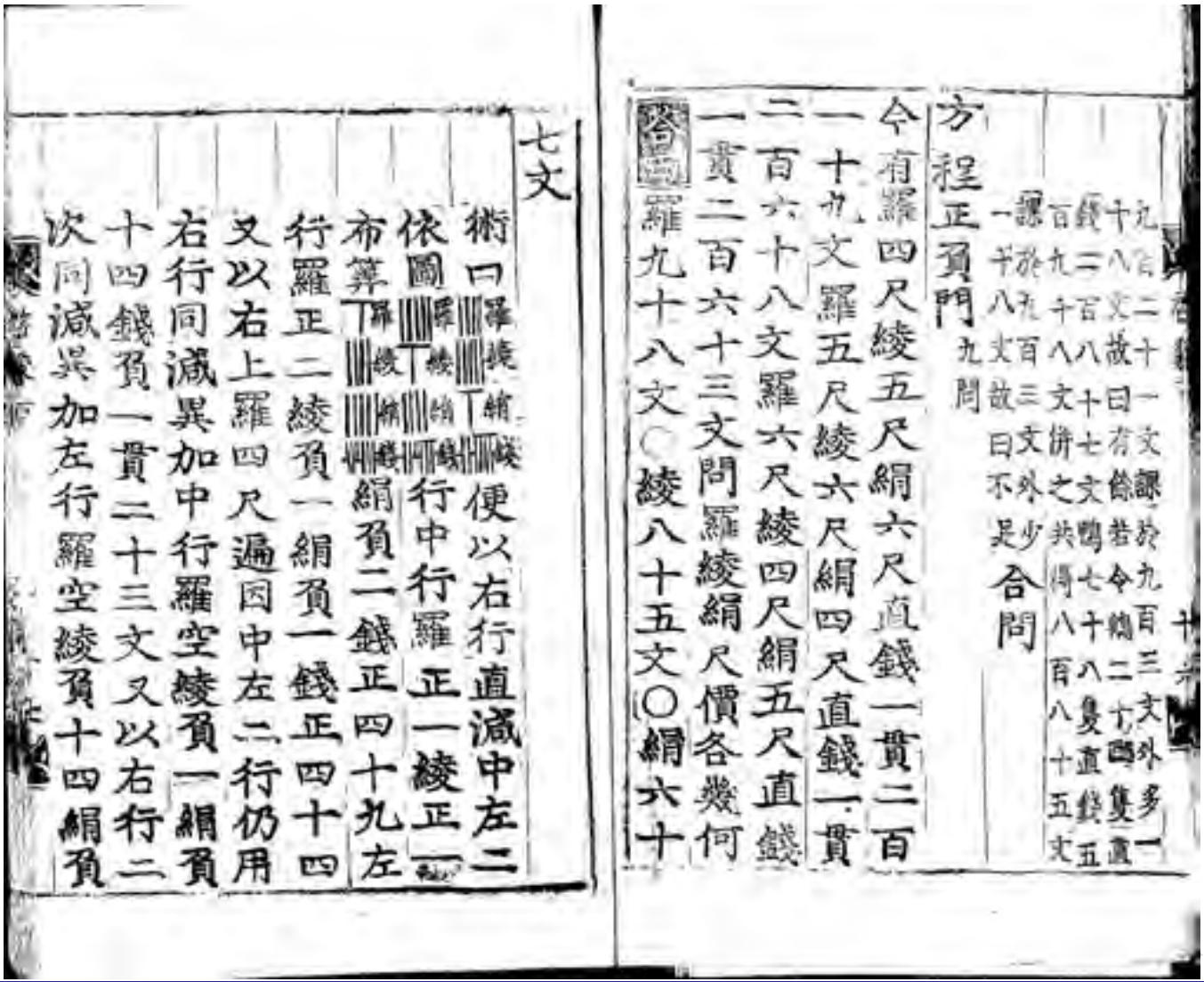


Figure: The letters A, B, and C represent unknowns, quantities that were originally written in Chinese characters. 巾 is a simplified form of the Chinese character 冪 (power) and A 巾 means A^2, 三 is the Chinese character for 3 and B 3 means B^4. B 再 means B^3. In Wasan and classical Chinese mathematics, B^4 was expressed by multiplying B by B three times.

Suanxue Qimeng



Seki's Elimination Theory

From the following pair of equations

$$f(x,y) = a_0(x) + a_1(x)y + a_2(x)y^2 + \cdots + a_n(x)y^n = 0,$$
$$g(x,y) = b_0(x) + b_1(x)y + b_2(x)y^2 + \cdots + b_n(x)y^n = 0.$$

Seki constructed a system of n transformed equations (Kanshiki) of degree $n-1$ in y:

$$F_1(x,y) = 0$$
$$F_2(x,y) = 0$$
$$\cdots\cdots$$
$$\cdots\cdots$$
$$F_n(x,y) = 0$$

Kanshiki

$F_k(x, y)$ is defined inductively:

$$\begin{aligned} F_1(x, y) &= a_n g(x, y) - b_n f(x, y) \\ &= a_n b_0 - b_n a_0 + \cdots + (a_n b_{n-1} - b_n a_{n-1}) y^{n-1}. \end{aligned}$$

$$\begin{aligned} F_2(x, y) &= a_{n-1} g(x, y) - b_{n-1} f(x, y) + F_1(x, y) y, \\ &\cdots\cdots \\ F_k(x, y) &= a_{n-k+1} g(x, y) - b_{n-k+1} f(x, y) + F_{k-1}(x, y) y, \\ &\cdots\cdots \\ F_n(x, y) &= a_1 g(x, y) - b_1 f(x, y) + F_{n-1}(x, y) y \end{aligned}$$

Kanshiki

$$F_k(x,y) = (a_n(x)y^{k-1} + a_{n-1}(x)y^{k-2} + \cdots + a_{n-k+1}(x))g(x,y),$$
$$- (b_n(x)y^{k-1} + b_{n-1}(x)y^{k-2} + \cdots + b_{n-k+1}(x))f(x,y)$$

$$F_1(x,y) = a_{11} + a_{12}y + a_{13}y^2 + \cdots + a_{1n}y^{n-1} = 0$$
$$F_2(x,y) = a_{21} + a_{22}y + a_{23}y^2 + \cdots + a_{2n}y^{n-1} = 0$$
$$\cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots$$
$$\cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots$$
$$F_n(x,y) = a_{n1} + a_{n2}y + a_{n3}y^2 + \cdots + a_{nn}y^{n-1} = 0$$

Here, Seki encountered the determinant.

Chikushiki-Kojō

Seki provided two different methods for defining the determinant. He called the first method "Chikushiki-Kojō", which involves multiplying each polynomial $F_k(x, y)$ by a certain polynomial in x and adding the product to obtain a polynomial in x. Specifically, he sought $A_k(x)$ such that

$$\sum_{k=1}^{n} A_k(x) F_k(x, y) = R(x)$$

where $R(x) = 0$ is the resultant for the original pair of equations. Seki seemes to have found $A_k(x)$ inductively based on n.

For $n = 2$,

$$a_{22}F_1(x, y) - a_{12}F_2(x, y) = a_{11}a_{22} - a_{12}a_{21}.$$

For $n = 3$ let us consider F_2 and F_3, delete the 0th-degree terms, and divide them by y to get

$$a_{21} + a_{22}y = 0$$
$$a_{31} + a_{32}y = 0.$$

The resultant of these equations is $A_1(x)$. To find $A_2(x)$ choose F_1 and F_3 to get

$$a_{11} + a_{12}y = 0$$
$$a_{31} + a_{32}y = 0.$$

Then, the negative of the resultant of these equations is $A_2(x)$.

Also from F_1 and F_2 we get
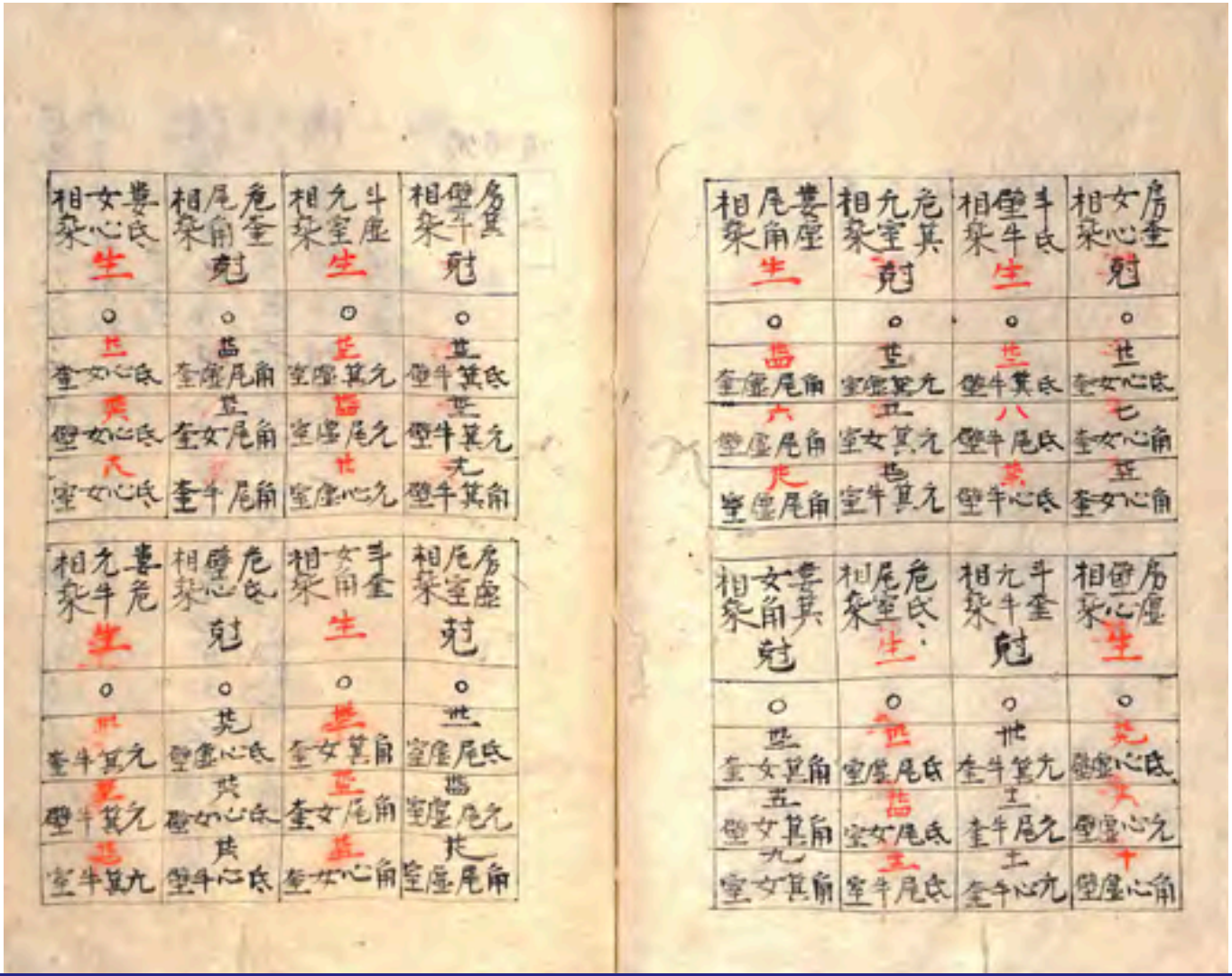
$$a_{11} + a_{12}y = 0$$
$$a_{21} + a_{22}y = 0.$$

The resultant of these equations is $A_3(x)$. Seki calculated all the terms appearing in $A_1(x)F_1(x, y) + A_2(x)F_2(x, y) + A_3(x)F_3(x, y)$ to show that this is a polynomial in x, and that the coefficients of y and y^2 vanish.

Seki *Kaifukudai no Hō*



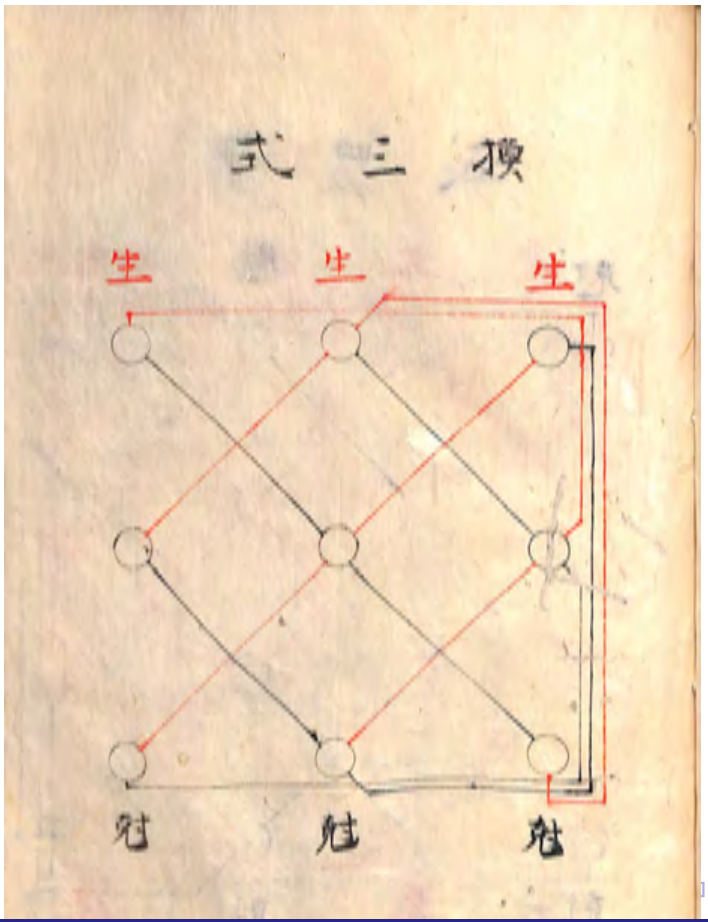
Seki *Kaifukudai no Hō*



Seki's second method

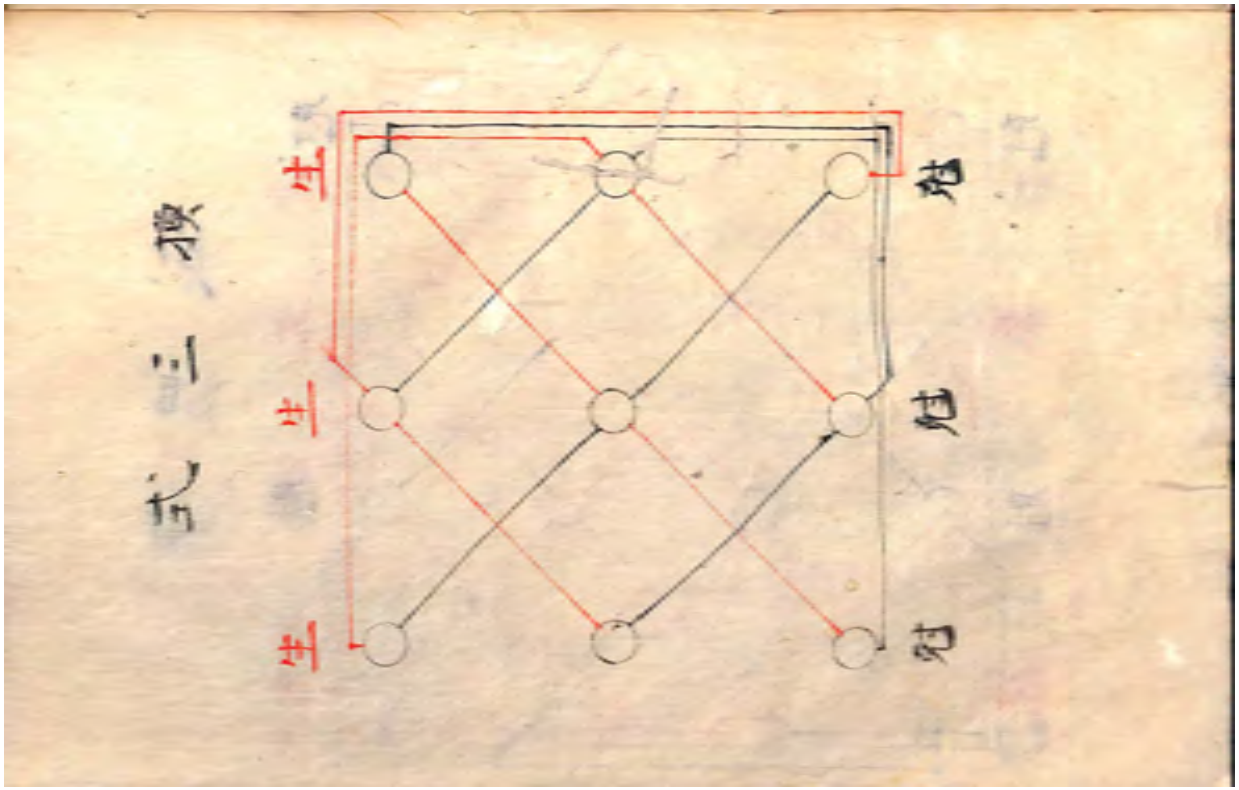
Kōshiki-Shajō

Shajō : Oblique multiplication

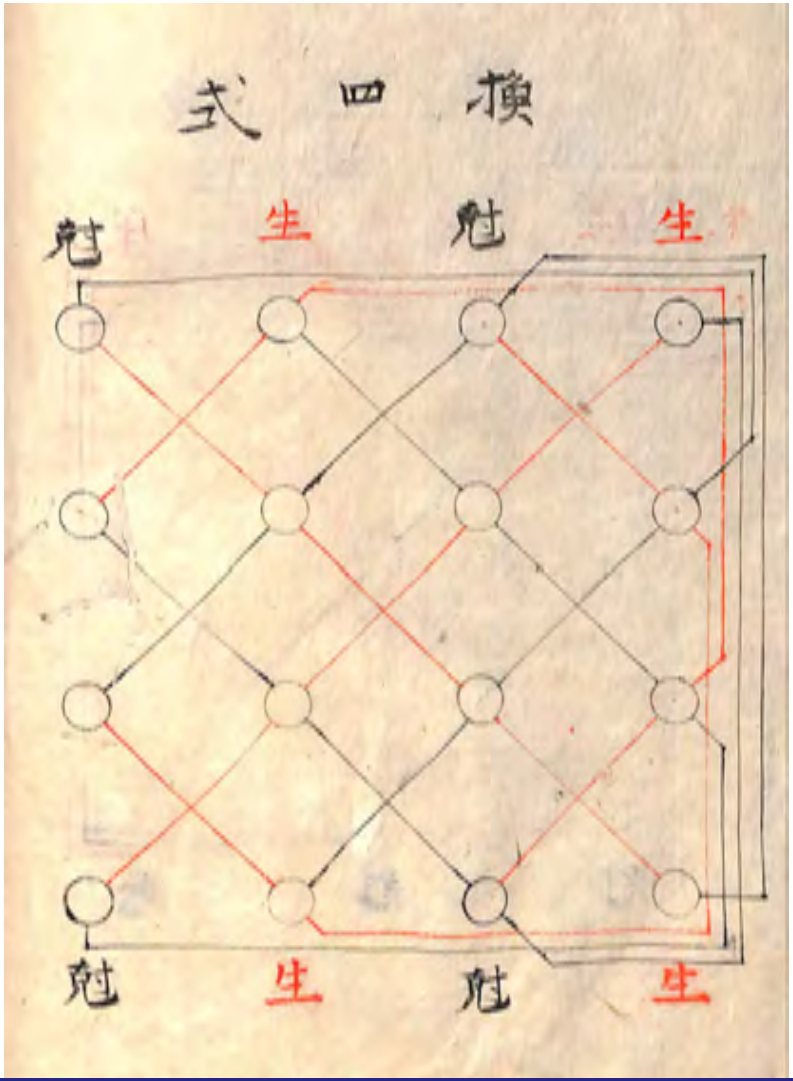


Shajō(oblique multiplication)

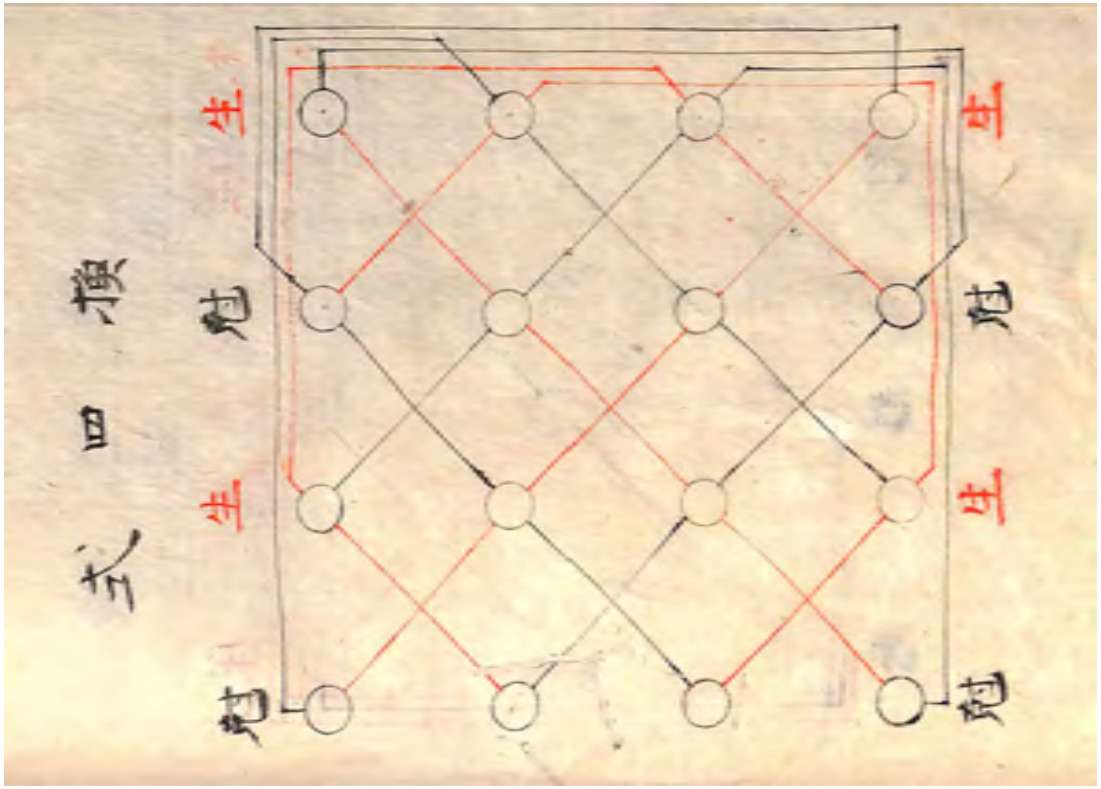
Sarrus's method



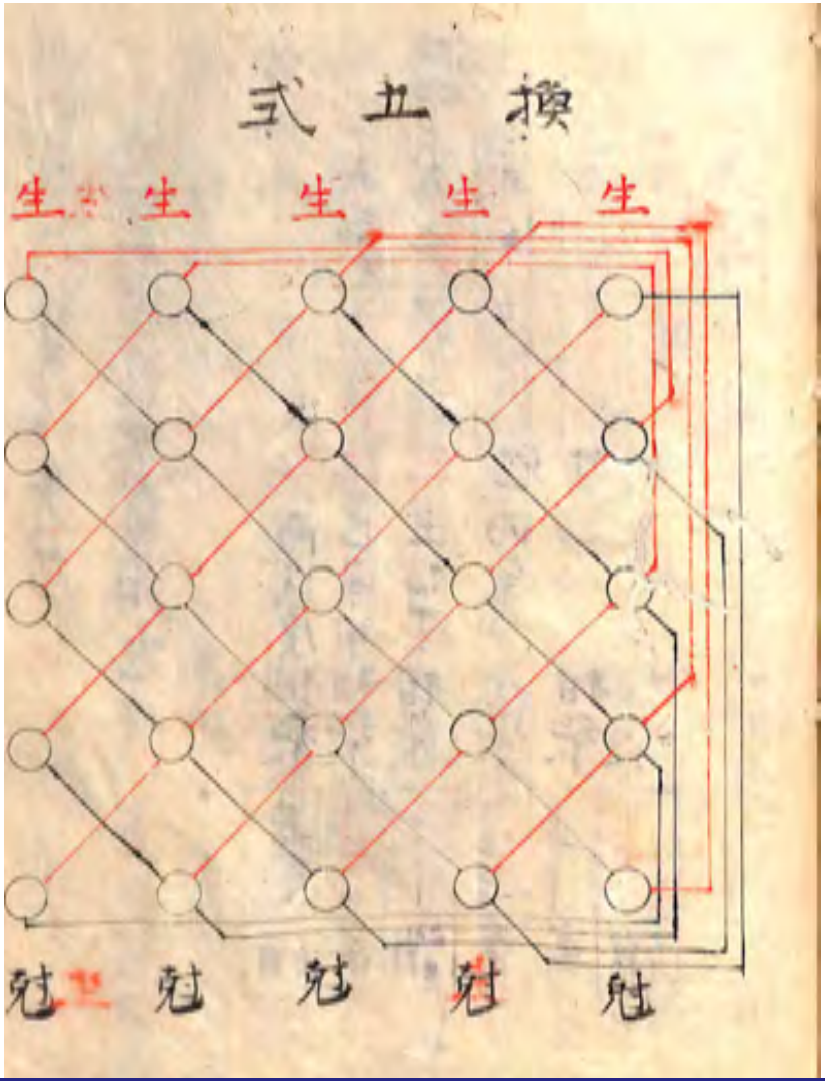
Shajō (oblique multiplication)



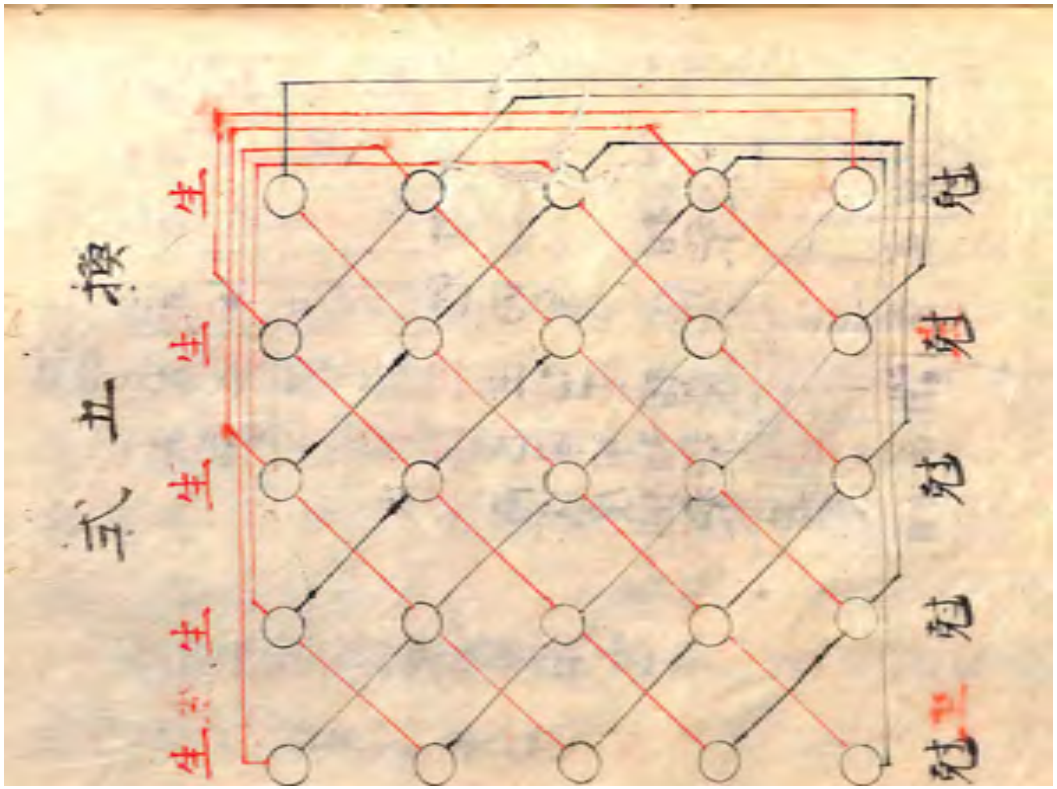
Shajō (oblique multiplication)



Shajō (oblique multiplication)



Shajō (Oblique multiplication)



In the $n \equiv 1 \pmod 4$ case the signs of the left and right oblique multiplications are always plus. Sugano and Ishikuro corrected this mistake in 1798.

$n = 4$

$$S\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$$

$$= a_{11}a_{22}a_{33}a_{44} - a_{21}a_{32}a_{43}a_{14} + a_{31}a_{42}a_{13}a_{24} - a_{41}a_{12}a_{23}a_{34}$$

$$+ a_{14}a_{23}a_{32}a_{41} - a_{24}a_{33}a_{42}a_{11} + a_{34}a_{43}a_{12}a_{21} - a_{44}a_{13}a_{22}a_{31}.$$

$$S(1, j_2, j_3, j_4) = S\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{j_2 1} & a_{j_2 2} & a_{j_2 3} & a_{j_2 4} \\ a_{j_3 1} & a_{j_3 2} & a_{j_3 3} & a_{j_3 4} \\ a_{j_4 1} & a_{j_4 2} & a_{j_4 3} & a_{j_4 4} \end{pmatrix} \qquad \{j_2, j_3, j_4\} = \{2, 3, 4\}$$

Kōshiki(rearranging the order of equations)

Seki showed

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} = S(1,2,3,4) + S(1,3,4,2) + S(1,4,2,3)$$

$n = 4$

Seki's Kōshiki: $(1,2,3,4)$, $(1,3,4,2)$, $(1,4,2,3)$.

$(1,2,3) \Longrightarrow (2,3,4) \Longrightarrow (3,4,2) \Longrightarrow (4,2,3)$,

$5 \equiv 2 \pmod 3$

Today's interpretation

D_n : a subgroup of S_n generated by the cyclic permutation $(1\,2\,3\cdots n)$ and

$$\omega_n = \begin{pmatrix} 1 & 2 & 3 & \cdots & n-1 & n \\ n & n-1 & n-2 & \cdots & 2 & 1 \end{pmatrix}.$$

$$S(1,2,\ldots,n) = \sum_{\sigma \in D_n} \mathrm{sgn}\,\sigma\, a_{\sigma(1)1} a_{\sigma(2)2} \cdots a_{\sigma(n)n}.$$

Kōshiki $\iff$ the right coset S_n/D_n

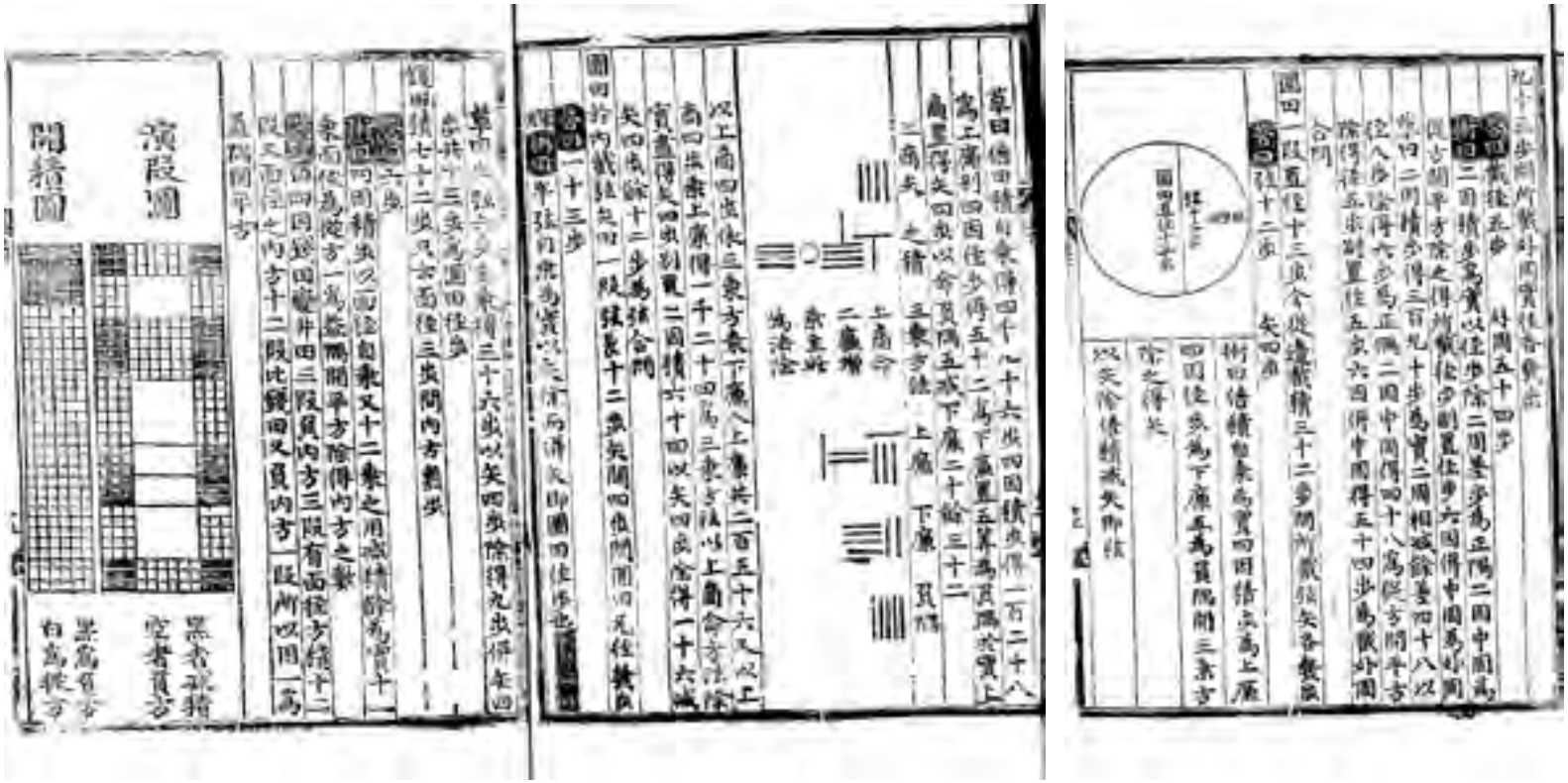
$$(1, j_2, j_3, \ldots, j_n) \longleftrightarrow \begin{pmatrix} 1 & 2 & 3 & \ldots & n \\ 1 & j_2 & j_3 & \ldots & j_n \end{pmatrix} D_n$$

Naoki Osada: Determinant by Seki Takakazu from the group-theoretic viewpoint, pp. 83–99, in Mathematics of Takebe Katahiro and History of Mathematics in East Asia, edited by Ogawa & Morimoto, Advanced in Pure Mathematics, vol.79(2018), Math. Soc. Japan.

The Seki-Horner Method

- Jia Xian, who lived during the Northern Song dynasty in the 11th century, discovered a numerical method for finding the positive roots of equations of any degree.
- This method is essentially equivalent to today's Ruffini-Horner method(synthetic division).
- It can be found in the *Yang Hui Sanfa*.

Yang Hui Sanfa



The problem is related to the unsolved tenth problem of the *Jinkoki*. Yang Hui showed how to solve the equation $-5x^4 + 52x^3 + 128x = 4096$ numerically by using counting rods.

The Seki-Horner Method

- Jia Xian used equations in the form $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x = A$.
- Qin Jiushao (1208–1268) of the Southern Song dynasty rewrote equations in the form $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x - A = 0$ making it identical to today's method of synthetic division.
- However, only positive roots were considered, and numerical solutions were found one digit at a time, starting with the highest place value.
- Qin Jiushao's book *Shushu Jinyuan* was first printed in China in 1842; prior to that, it had been transmitted in manuscript form. There is no evidence that any manuscripts of this work were brought to Japan during the Edo period.

The Seki-Horner Method

- The mathematical achievements of Seki Takakazu and Qin Jiushao overlap to some extent. This is because both of them developed new branches of mathematics based on virtually the same mathematical foundations.
- Takakazu Seki completed a method for finding numerical solutions to equations using synthetic division.
- Seki also considered negative roots. Even when finding positive roots, he freely used both positive and negative numbers in synthetic division to find numerical solutions.

Seki's Method

$$f(x) = x^3 + 2x^2 + 3x - 9 = 0$$

First, use 1 in synthetic division.

1)	1	2	3	-9
		1	3	6
1)	1	3	6	$\boxed{-3}$
		1	4	
1)	1	4	$\boxed{10}$	
		1		
	$\boxed{1}$	$\boxed{5}$		

By setting $x_1 = x - 1$, the equation $f(x) = 0$ transforms into

$$f_1(x_1) = x_1^3 + 5x_1^2 + 10x_1 - 3 = 0$$

Seki's Method

$$f_1(x_1) = x_1^3 + 5x_1^2 + 10x_1 - 3 = 0$$

0.2)	1	5	10	-3
		0.2	1.04	2.208
0.2)	1	5.2	11.04	$\boxed{-0.792}$
		0.2	1.08	
0.2)	1	5.4	$\boxed{12.12}$	
		0.2		
	$\boxed{1}$	$\boxed{5.6}$		

Setting $x_2 = x_1 - 0.2$ transforms $f_1(x_1) = 0$ into

$$x_2^3 + 5.6x_2^2 + 12.12x_2 - 0.792 = 0.$$

Seki's Method

$$x_2^3 + 5.6x_2^2 + 12.12x_2 - 0.792 = 0.$$

0.06)	1	5.6	12.12	-0.792
		0.06	0.3396	0.747576
0.06)	1	5.66	12.4596	$\boxed{-0.044424}$
		0.06	0.3432	
0.06)	1	5.72	$\boxed{12.8028}$	
		0.06		
	$\boxed{1}$	$\boxed{5.78}$		

Setting $x_3 = x_2 - 0.06$ gives $f_3(x_3) = 0$ in the form

$$x_3^3 + 5.78x_3^2 + 12.8028x_3 - 0.044424 = 0$$

Seki's Method

$$f_3(x_3) = x_3^3 + 5.78x_3^2 + 12.8028x_3 - 0.044424 = 0$$

Since the constant term is extremely close to zero, we can take

$$0.044424/12.8028 = 0.0034698\cdots$$

as an approximate solution to $f_3(x_3) = 0$. Therefore, we can take

$$1 + 0.2 + 0.06 + 0.0034698 = 1.2634698$$

as an approximate solution to the original equation.

The Seki-Bernoulli Numbers

$$s_p(n) = 1^p + 2^p + \cdots + (n-1)^p + n^p$$

- Seki and Jakob Bernoulli independently introduced the Seki-Bernoulli numbers and showed Faulhaber's formula:

$$(p+1)s_p(n) = \sum_{k=0}^{p} \binom{p+1}{k} B_k n^{p+1-k}$$

- B_k is defined inductively by the relation

$$p+1 = \sum_{k=0}^{p} \binom{p+1}{k} B_k.$$

The Seki-Bernoulli Numbers

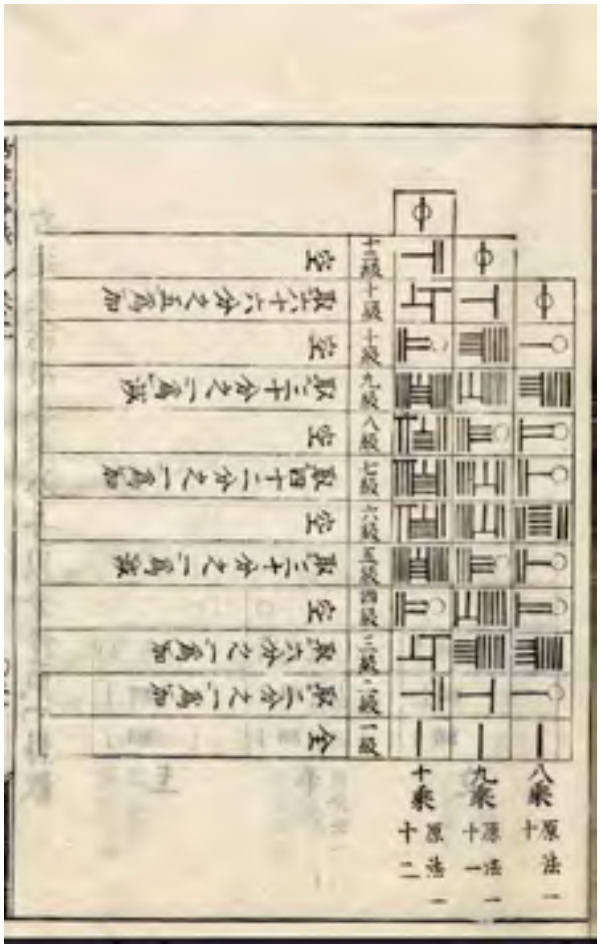
Katsuyo Sanpo, 1712.





Seki-Bernoulli Numbers

$$(x+1)^{p+1}-1$$





S_p												deg $S_p(n)$
$2S_1$	1	1										2
$3S_2$	1	$\frac{3}{2}$	$\frac{1}{2}$									3
$4S_3$	1	2	1									4
$5S_4$	1	$\frac{5}{2}$	$\frac{5}{3}$	0	$-\frac{1}{6}$							5
$6S_5$	1	3	$\frac{5}{2}$	0	$-\frac{1}{2}$							6
$7S_6$	1	$\frac{7}{2}$	$\frac{7}{2}$	0	$-\frac{7}{6}$	0	$\frac{1}{6}$					7
$8S_7$	1	4	$\frac{14}{3}$	0	$-\frac{7}{3}$	0	$\frac{2}{3}$					8
$9S_8$	1	$\frac{9}{2}$	6	0	$-\frac{21}{5}$	0	2	0	$-\frac{3}{10}$			9
$10S_9$	1	5	$\frac{15}{2}$	0	-7	0	5	0	$-\frac{3}{2}$			10
$11S_{10}$	1	$\frac{11}{2}$	$\frac{55}{6}$	0	-11	0	11	0	$-\frac{11}{2}$	0	$\frac{5}{6}$	11
$12S_{11}$	1	6	11	0	$-\frac{33}{2}$	0	22	0	$-\frac{33}{2}$	0	5	12

The coefficients of $S_p(n)$ are listed in descending order.

By comparing the binomial coefficients of $(x+1)^{p+1}-1$ and $(p+1)S_p(n)$ Seki found the Seki-Bernouli numbers.

Ars Conectandi 1713



PARS SECUNDA. 97

$$\infty \frac{n^6-6n^5+11n^4-6n}{6}, \text{ erit utique } \frac{n^5-6n^4+11n^3-6n^2}{5}, \text{ hoc est,}$$

$$f\frac{1}{2}n^5-\beta n^4+f'\frac{1}{2}n^3-f\frac{1}{2}n^2 \infty \frac{n^6-6n^5+11n^4-6n}{6}, \text{ indeque } f\frac{1}{2}n^5 \infty$$

$$\frac{n^6-6n^5+11n^4-6n}{6}+\beta n^4-f'\frac{1}{2}n^3+f\frac{1}{2}n^2. \text{ Et quoniam per modo in-}$$

venta $\beta n \infty \frac{1}{5}n^5+\frac{1}{2}n^4+\frac{1}{3}n^3$; nec non $f'\frac{1}{2}n$ sive $\frac{1}{2}f n \infty \frac{1}{10}n^2+\frac{1}{12}n$,

& $f\frac{1}{2} \infty n$; hinc factâ horum substitutione emerget $f\frac{1}{2}n^5 \infty$

$$\frac{n^6-6n^5+11n^4-6n^3}{6}+\frac{1}{5}n^5+\frac{1}{2}n^4+\frac{1}{3}n^3-\frac{1}{2}n^4+n^3-\frac{1}{2}n^2+\frac{1}{3}n^2 \infty$$

$\frac{1}{6}n^6+\frac{1}{30}n^5+\frac{1}{12}n^4$, cujusque proin sextuplum f^{6} (summa cubo-

rum) $\infty \frac{1}{5}n^6+\frac{1}{5}n^5+\frac{1}{2}n^4$. Atque sic porrò ad altiores gradatim

potestates pergere, levique negotio sequentem adornare lateculum

licet:

Summæ Potestatum.

$$\begin{aligned}
f\frac{1}{2} &\infty \tfrac{1}{2}n^2+\tfrac{1}{2}n.\\
f\frac{1}{2}n &\infty \tfrac{1}{3}n^3+\tfrac{1}{2}n^2+\tfrac{1}{6}n.\\
f\frac{1}{2}n^2 &\infty \tfrac{1}{4}n^4+\tfrac{1}{2}n^3+\tfrac{1}{4}n^2.\\
f\frac{1}{2}n^3 &\infty \tfrac{1}{5}n^5+\tfrac{1}{2}n^4+\tfrac{1}{3}n^3-\tfrac{1}{30}n.\\
f\frac{1}{2}n^4 &\infty \tfrac{1}{6}n^6+\tfrac{1}{2}n^5+\tfrac{5}{12}n^4-\tfrac{1}{12}n^2.\\
f\frac{1}{2}n^5 &\infty \tfrac{1}{7}n^7+\tfrac{1}{2}n^6+\tfrac{1}{2}n^5-\tfrac{1}{12}n^3+\tfrac{1}{12}n.\\
f\frac{1}{2}n^6 &\infty \tfrac{1}{8}n^8+\tfrac{1}{2}n^7+\tfrac{7}{12}n^6-\tfrac{7}{24}n^4+\tfrac{1}{12}n^2.\\
f\frac{1}{2}n^7 &\infty \tfrac{1}{9}n^9+\tfrac{1}{2}n^8+\tfrac{7}{12}n^7-\tfrac{7}{24}n^5+\tfrac{1}{12}n^3.\\
f\frac{1}{2}n^8 &\infty \tfrac{1}{10}n^{10}+\tfrac{1}{2}n^9+\tfrac{2}{3}n^8-\tfrac{7}{15}n^6+\tfrac{2}{3}n^4-\tfrac{1}{30}n^2.\\
f\frac{1}{2}n^9 &\infty \tfrac{1}{11}n^{11}+\tfrac{1}{2}n^{10}+\tfrac{3}{4}n^9-\tfrac{7}{10}n^7+1n^5-\tfrac{3}{4}n^3+\tfrac{5}{66}n.
\end{aligned}$$

Quin imò qui legem progressionis inibi attentius inspexerit, eundem etiam continuare poterit absq; his ratiocinationum ambagibus: Sumtâ enim c pro potestatis cujuslibet exponente, & summa omnium n^c seu $f\frac{1}{2}n^c \infty \frac{1}{c+1}n^{c+1}+\frac{1}{2}n^c+\frac{c}{12}An^{c-1}+\frac{c.c-1.c-2}{2.3.4}Bn^{c-3}+$

$\frac{c.c-1.c-2.c-3.c-4}{2.3.4.5.6}Cn^{c-5}+\frac{c.c-1.c-2.c-3.c-4.c-5.c-6}{2.3.4.5.6.7.8}Dn^{c-7}+$ &c. ita deinceps, exponentem potestatis ipsius n continuè minuendo binario, quousque perveniatur ad n vel nn. Litteræ capitales A, B, C, D &c. ordine denotant coefficientes ultimorum terminorum pro fn, fn^3, fn^5, fn^7 &c. nempe A $\infty \frac{1}{2}$, B $\infty -\frac{1}{6}$,

N



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$\infty -\frac{1}{30}$, C $\infty \frac{1}{42}$, D $\infty -\frac{1}{30}$. Sunt autem hi coefficientes ita comparati, ut singuli cum cæteris sui ordinis coefficientibus complere debeant unitatem; sic D valere debuisse $-\frac{1}{30}$, quia $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{42}+\frac{1}{30}(+D) \infty 1$. Hujus lateculi beneficio inveni primi quadratorum horæ reperi, quòd potestates decimæ sive quadrato-cubicæ mille primorum numerorum ab unitate in summam collecta efficiunt

91409924241424243424341924242500.

E quibus apparet, quàm inutilis censenda sit opera Rivallii Bal. Ibaldi, quam conscribendo tam spisso volumini Arithmeticæ suæ Infinitorum impendit, ubi nihil præstitit aliud, quàm ut primarum tantum sex potestatum summas (partem ejus quod unicâ nos conjecturi formæ paginâ) immenso labore demonstratas exhiberet.

Antequam caput hoc finiamus, juvabit adhuc indicare solet, quomodo suppositis iis, quæ de seriebus figuratis ostensa sunt, possint quævis etiam aliæ series figuratarum analogæ (quæ scil. differentias suas primas, secundas, tertias &c. æquales habent, adeoque ex continua additione terminorum alicujus seriei æqualium generantur) ad homologas figuratas reduci, ac proinde summari, vel positorum ipsarum termini invenirí. Sit series quævis æqualium D, ex cujus additione nascatur series jC, & ex hujus additione series B, & ex hujus eadem collectione series A, summis ad arbitrium primis seriei terminis d, i, k, z. Vocabitur series A figuratarum analoga, cujus differentiæ primæ constituunt seriem B, secundæ seriem C,

D	C	B	A.	
d	c	b	a	
d	$c+d$	$b+c$	$a+b$	
d	$c+2d$	$b+2c+d$	$a+2b+c$	
d	$c+3d$	$b+3c+3d$	$a+3b+3c+d$	
d	$c+4d$	$b+4c+6d$	$a+4b+6c+4d$	
d	$c+5d$	$b+5c+10d$	$a+5b+10c+10d$	

tertiæ seriem D, &c. Et quoniam apparet, seriem A componi ex seriebus unitatum $1, 1, 1, 1$ &c. lateralium $1, 2, 3, 4$. &c. trigonalium $1, 3, 6, 10$, &c. pyramidalium $1, 4, 10, 20$. &c. in primos differentiarum terminos a, b, c, d, seorsim ductis, quarumque omnium posterm termini & summæ per ante dicta habentur, ipsius quoque

Geometric Series

Infinite geometric series

$$\sum_{n=0}^{\infty} ar^k = \frac{a}{1-r}$$

Seki was the first mathematician in the realm of classical Chinese mathematics to consider the sum of an infinite geometric series with a common ratio r.

$$a - \sum_{n=1}^{\infty} ar^k = \frac{a-2ar}{1-r}$$

Yang Hui Sanfa

Yang Hui Sanfa Vol.1 Problem 6 describes the calculation of

$$\frac{n}{3333} = \frac{n}{10^4} \times \frac{3333 \times 3 + 1}{3333} = \frac{3n}{10^4} + \frac{n}{10^4} \times \frac{1}{3333}$$

If we continue the calculation, we get

$$\frac{3n}{10^4} + \frac{3n}{10^8} + \frac{3n}{10^{12}} + \cdots$$

We also find the calculation:

$$\frac{n}{1666} = \frac{n}{10^4} \cdot \frac{1666 \times 6 + 4}{1666} = \frac{6n}{10^4} + \frac{4n}{10^4} \cdot \frac{1}{1666},$$

which leads to

$$\frac{6n}{10^4} + \frac{24n}{10^8} + \frac{96n}{10^{12}} + \cdots$$

Calculation of π

Seki's theories often originated from extensive calculations. Seki calculated the perimeter σ_n of a regular 2^n-gon inscribed in a circle with diameter 1 to 20 decimal places. His results were as follows:

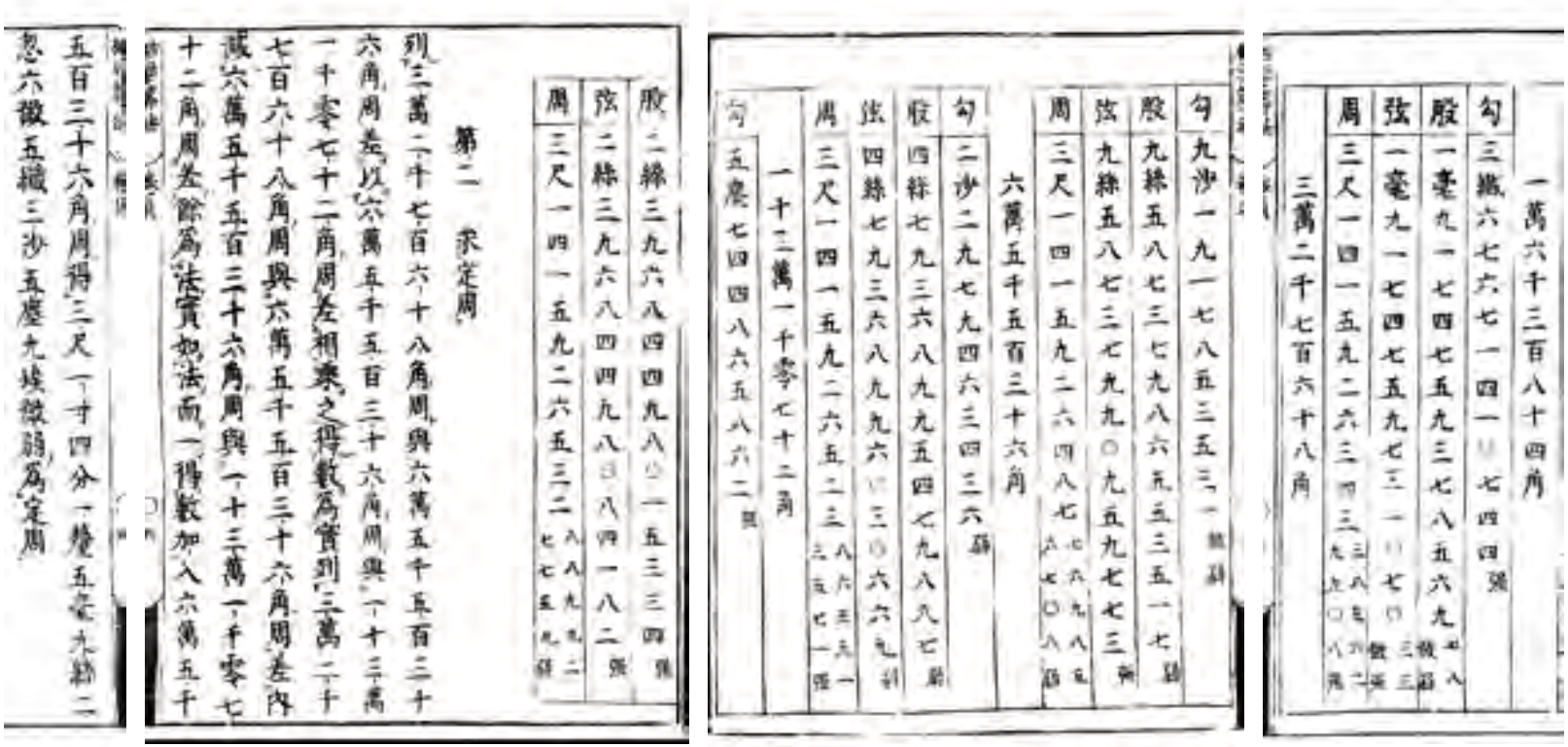
$$\sigma_{13} = 3.14159257658487266 68$$
$$\sigma_{14} = 3.14159263433856299 08$$
$$\sigma_{15} = 3.14159264877698567 08$$
$$\sigma_{16} = 3.14159265238659135 71$$
$$\sigma_{17} = 3.14159265328899277 59$$

Then Seki claimed that

$$\sigma_{16} + \frac{(\sigma_{16} - \sigma_{15})(\sigma_{17} - \sigma_{16})}{(\sigma_{16} - \sigma_{15}) - (\sigma_{17} - \sigma_{16})}$$

gives a better approximation 3.14159265359 of π.

Katsuyo Sanpo vol.4



Aitken Δ^2 Method

- The infinite series
  $\pi = \sigma_{15} + (\sigma_{16} - \sigma_{15}) + (\sigma_{17} - \sigma_{16}) + (\sigma_{18} - \sigma_{17}) + \cdots$ can be approximated by σ_{15} + a geometric series:

  $$\sigma_{15}+(\sigma_{16}-\sigma_{15})+(\sigma_{16}-\sigma_{15})r+(\sigma_{16}-\sigma_{15})r^2+(\sigma_{16}-\sigma_{15})r^3+\cdots.$$

  with

  $$r = \frac{(\sigma_{17} - \sigma_{16})}{(\sigma_{16} - \sigma_{15})}$$

  Then

  $$\sigma_{15} + (\sigma_{16} - \sigma_{15}) + \frac{(\sigma_{16} - \sigma_{15})r}{1 - r}$$

  approximates π.
- Seki used a similar method to calculate the length of an arc and the volume of a sphere.

Conclusion

- Seki changed the concept of mathematics in the realm of classical Chinese mathematics.
- He emphasized the importance of a general mathematical theory and developed several theories of his own.
- For instance, he completed the elimination theory of systems of equations (the theory of resultants) over 80 years before Western mathematicians did, and he applied it to the study of equations. (Note that equations were not a subject of mathematical study in classical Chinese mathematics. They were merely a tool for solving problems.)
- However, Seki's mathematics lacked the concept of proof. Without proofs, general theories were difficult to understand, so later Japanese mathematicians could grasp his mathematical results but not his ideas about general theories.